One Step towards Bridging the Gap between Theory and Practice in Moldable Task Scheduling with Precedence Constraints

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Motivation - Divergence of Scheduling Research

- parallel machine scheduling
  - complex architecture of parallel machines / software (OS)
  - (almost) impossible to know much about the problem
    - job sizes (execution times)
    - release dates, etc.

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  - "an interesting problem, we need novel insights"
    - let us use a simplistic model
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- practitioners
  - "I forgot my Turing machine today"
  - "How do I adapt your FPTAS to 3 levels of cache on 200,000 NUMA cores interconnected via 6D torus network under a certain workload and background system noise?"
  - "I also figure that your $O(n^{60})$ DP could be a little slow."
  - "But actually, I do not care about theoretical results. I will simply reinvent the wheel and sell it as breakthrough."

Situation

Theoretical Results .... VOID .... Practical Results

The Problem

- notation follows "Scheduling for Parallel Processing" by Drozdowski [Dro09]
- types of parallel tasks
  - rigid
  - moldable
  - malleable
- precedence constraints between n tasks
  - direct acyclic graph (DAG)
- schedule n moldable tasks on m identical processors
- in Graham's 3-field notation
  - $P|any, NdSub, prec|C_{\max}$ and $P|any, prec|C_{\max}$
  - any - moldable tasks
  - $NdSub$ - nondecreasing sublinear speedup
  - $prec$ - precedence constraints

Common Assumptions - NdSub ?

- PROC-TIME-NON-INCREASING
  The processing time $p(l)$ of a moldable task J is non-increasing in the number l of the processors allotted to it, that is, $p(l) \leq p(l')$, for $l \geq l'$;

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- PROC-TIME-STRICTLY-DECREASING
  The processing time $p(l)$ of a moldable task is strictly decreasing in the number l of the allocated processors: $p(l) < p(l')$, for $l > l'$.

Common Assumptions - NdSub ? (cont'd)

- SPEEDUP-CONCAVE "The first restriction is that all speedup functions are concave at least between 0 and the processor number $\hat{p}$ where the maximal speedup is reached." [SS12]

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- WORK-CONVEX-PROC-TIME
  The work function $w(p(l))$ is convex in the processing time $p(l)$.

Related Work - Lepère, Trystram, Woeginger [LTW02]

- assumptions
  1. PROC-TIME-NON-INCREASING
  2. WORK-NON-DECREASING
- decouple scheduling problem
- allotment problem (MT-ALLOTMENT)
  - Skutella's linear relaxation of discrete time-cost trade-off problem (DTCT)
  - 2 approximation
- mapping problem (MT-MAKESPAN)
  - Graham's list scheduling for parallel tasks
    - earliest possible task first
  - the proof is the trick here
  - construct a heavy path in the transitive closure of the DAG
- approximation ratio: $3 + \sqrt{5} \approx 5.23606$

Related Work - Jansen and Zhang [JZ06]

- same assumptions as [LTW02]
  1. PROC-TIME-NON-INCREASING
  2. WORK-NON-DECREASING
- provide linear program formulation
- kept mapping function
- the trick here is to find the right rounding parameter ρ
  - round fractional solution to feasible allotment
- approximation ratio: ≈ 4.730598

Related Work - Jansen and Zhang [JZ12]

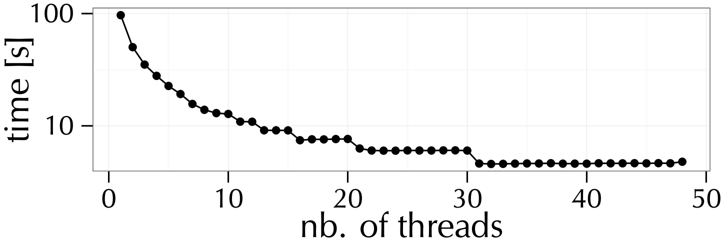
- assumptions:
  1. PROC-TIME-NON-INCREASING
  2. WORK-NON-DECREASING
  3. SPEEDUP-CONCAVE
- allotment: reformulate the LP
  - essence: use a variable indicating that a job is (fractionally) allocated to l processors
  - constraint: $\sum_{l=1}^{m} x_{j,l} = 1$
  - interesting here: at most two $x_{j,l}$ are non-zero and adjacent
- mapping step unchanged
- approximation ratio: ≈ 3.291919

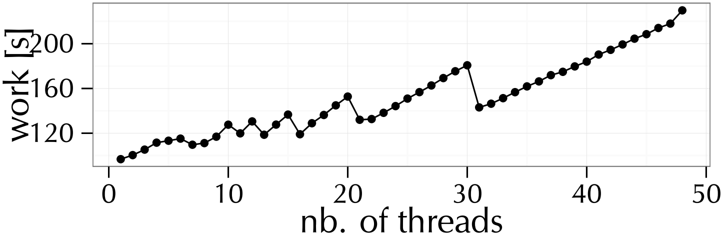
Related Work - Chen and Chu [CC13]

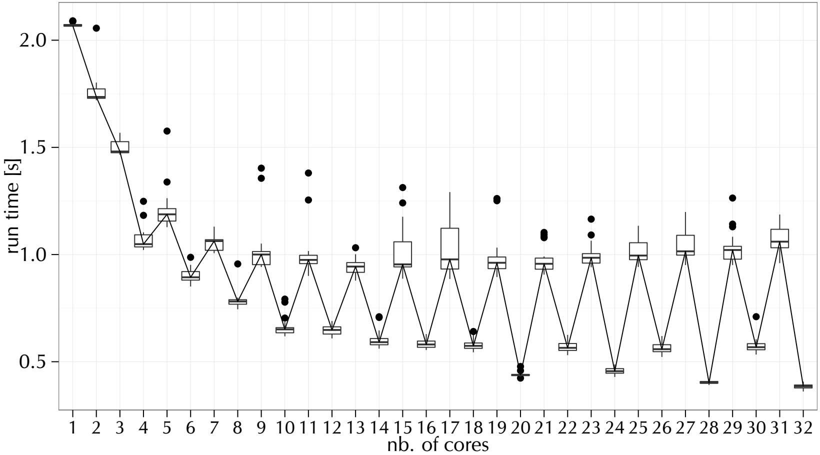
- an algorithm (heavily) based on JZ06
- assumptions
  1. PROC-TIME-NON-INCREASING
  2. WORK-NON-DECREASING
  3. WORK-CONVEX-PROC-TIME
- allotment:
  - precompute the work of each possible allocation for all tasks
  - use this information to add constraints to LP (which later help in the rounding step)
- mapping unchanged
- approximation ratio: ≈ 3.4142
- then, adding
  1. PROC-TIME-STRICTLY-DECREASING
  2. leads to approximation ratio: 2.9549

Reality 1

NAS PB - LU benchmark (4 sockets, 48 cores, AMD Opteron 6168)







MPICH 3.0.4, 32x1 (process per node)

CPA13 heuristic

- based on CPA by Radulescu and van Gemund [RvG01]
- main purpose: schedule tasks with arbitrary speed-up functions
- allotment solution, ingredients:
  - consider only allocations which provide a relative runtime gain of $x\,\%$
  - force task parallel execution of tasks
  - iteratively add processors to tasks on critical path
  - until $L_{CP} < W/m$
- mapping solution
  - similar to list scheduling approach of Lepère, Trystram, Woeginger
  - but prioritize tasks by bottom level (length of path to sink)
  - but consider packing of tasks if estimated completion time is not increased (binary search to find possibly smaller allotment)

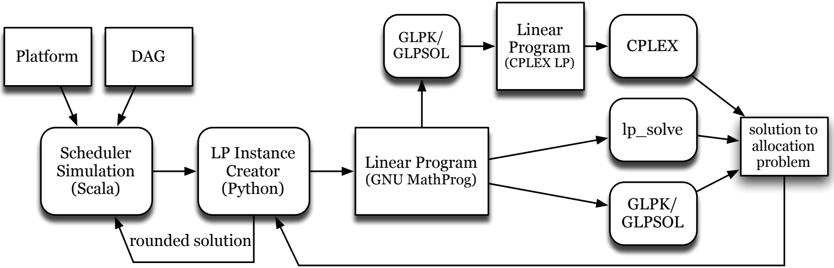
General Questions

- How good are "frequently cited" heuristics (e.g., CPA) compared to approximation algorithms?
- How fast are current LP solvers (e.g., CPLEX) for solving "practically relevant" problems?

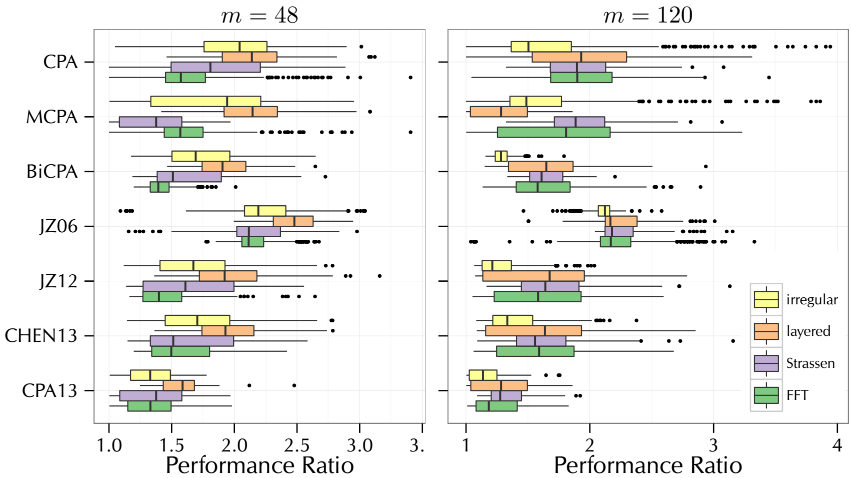
algorithm	allocation	mapping
CPA13	$O(nm(n+e))$	$O(n(\log n + m \log m) + e)$
JZ06	$O(LP(mn, n^2 + mn))$	$O(mn)$
JZ12	$O(LP(mn, n^2 + mn))$	$O(mn)$
Chen13	$O(LP(mn, n^2 + mn) + mn)$	$O(mn)$

- let us fulfill all assumptions
  - PROC-TIME-NON-INCREASING, WORK-NON-DECREASING, PROC-TIME-STRICTLY-DECREASING, SPEEDUP-CONCAVE, WORK-CONVEX-PROC-TIME

Experimental Setup

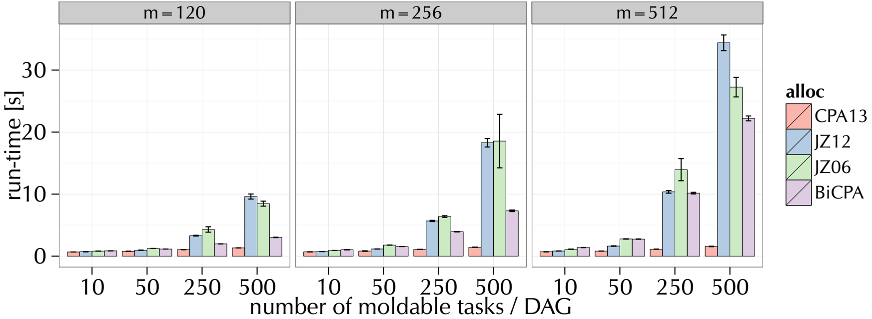


Simulation Results - Distribution of Makespans



performance ratio $\hat{=}\ C_{\text{max}}/LB$

Scalability



- Intel i7-3770 @ 3.40 GHz, 4 cores / 8 hardware threads
- CPLEX Studio 12.5.1 Linux x86-64
  - LP programs use all 4 cores

But can I be sure that the results are correct?

Missing Constraint - [JZ06]

$$
\begin{array}{lll}
\text{min} & C & \\
\text{such that} & 0 \le C_j \le L & \text{for all } j \\
& C_j + x_k \le C_k & \text{for all } j \text{ and } k \in \Gamma^+(j) \\
& x_j \le C_j & \text{for all } j \\
& x_j \le p_j(1) & \text{for all } j \\
& x_{j_i} \le x_j & \text{for all } j \text{ and } i = 1, \dots, m \\
& 0 \le x_{j_i} \le p_j(i) & \text{for all } j \text{ and } i = 1, \dots, m-1 \\
& x_{j_m} = p_j(m) & \text{for all } j \\
& \hat{w}_j(x_j) = \sum_{i=1}^{m} \bar{w}_{j_i}(x_{j_i}) & \text{for all } j \\
& P = \sum_{j=1}^{n} p_j(1) & \\
& \sum_{j=1}^{n} \hat{w}_j(x_j) + P \le W & \\
& L \le C & \\
& W/m \le C & \\
& \bar{w}_j(x_{j_m}) = 0 & \text{for all } j \\
& \bar{w}_{j_i}(x_{j_i}) = [W_j(i+1) - W_j(i)] \dfrac{p_j(i) - x_{j_i}}{p_j(i)} & \text{for all } j \text{ and } i = 1, \dots, m-1
\end{array}
$$

Missing Constraint (2) - [CC13]

- LP of Chen and Chu also misses same constraint
- for assumption PROC-TIME-STRICTLY-DECREASING paper provides another rounding procedure leading to smaller bound
  - problem: rounding procedure described in paper for strictly decreasing function produced very large $C_{\max}$'es (why?)
- run-time of this LP was huge
  - reason: LP uses a possibly different set of processor allocations for each moldable task (many more constraints)

Summary of Problems Occurred

- missing constraints in linear programs
  - very time-consuming to detect (at least for me)
- the problem of precision: floats/double
  - "64 Bit is finite"
  - problem generator
    - choose the execution time of tasks (for 1 proc) randomly and apply some strong strong scaling function
    - $0.00000000000001345 \not< 0.00000000000001345$

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- many sources of errors
  - me
  - DAG generator
  - platform generator
  - predictor for execution time of moldable tasks
  - translation of linear program in mathematical notation to AMPL/MathProg
  - parsing results from LP solver
  - implementation of mapping algorithm / simulator

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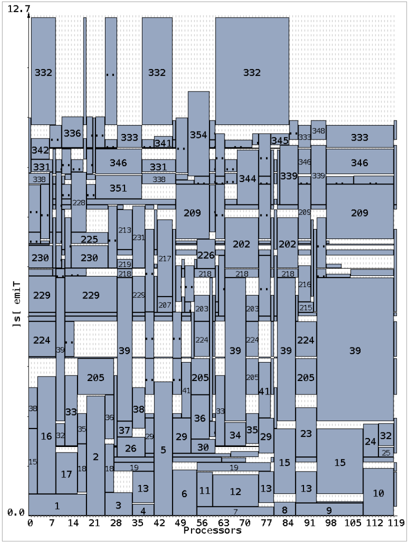
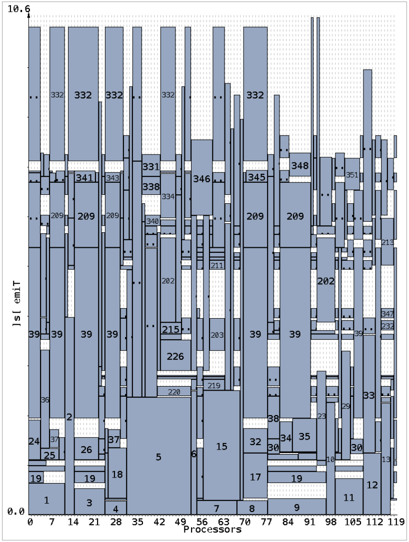
  - implementation of mapping algorithm / simulator
- consequence: debugging is a nightmare

Linear Program Solvers – Primal vs. Dual @ JZ06



`--primal`



`--dual`

Summary - What Did I Miss in this Study?

- help from theoreticians
- public database with codes (e.g., LPs)
- set of benchmarks
  - relevant problems (for DAGs of moldable tasks)
  - possibly optimal solutions for small instances
  - implementations of algorithms (give me the sources)
  - some example schedules produced by algorithms
- wish list (from the view of a practitioner)
  - Complexity results for scheduling problems
    - `http://www.informatik.uni-osnabrueck.de/knust/class/`
  - add implementation and test cases
- my source code is available upon request

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