One Step towards Bridging the Gap between Theory and Practice in Moldable Task Scheduling with Precedence Constraints

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parallel machine scheduling
  complex architecture of parallel machines / software (OS)
  (almost) impossible to know much about the problem
  job sizes (execution times)
  release dates, etc.
Motivation - Divergence of Scheduling Research

- parallel machine scheduling
  - complex architecture of parallel machines / software (OS)
  - (almost) impossible to know much about the problem
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- theoreticians
  - "an interesting problem, we need novel insights"
  - let us use a simplistic model
  - "I found an FPTAS for the simplistic model. I have a complicated 100-page proof. Problem solved."
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- practitioners
  - "I forgot my Turing machine today"
  - "How do I adapt your FPTAS to 3 levels of cache on 200,000 NUMA cores interconnected via 6D torus network under a certain workload and background system noise?"
  - "I also figure that your $O(n^{60})$ DP could be a little slow."
  - "But actually, I do not care about theoretical results. I will simply reinvent the wheel and sell it as breakthrough."
Theoretical Results ........ VOID ............ Practical Results
The Problem

- notation follows "Scheduling for Parallel Processing" by Drozdowski [Dro09]
- types of parallel tasks
  - rigid
  - moldable
  - malleable
- precedence constraints between \( n \) tasks
  - direct acyclic graph (DAG)
- schedule \( n \) moldable tasks on \( m \) identical processors
- in Graham’s 3-field notation
  - \( P|\text{any, NdSub, prec}|C_{\text{max}} \) and \( P|\text{any, prec}|C_{\text{max}} \)
  - \text{any} - moldable tasks
  - \text{NdSub} - nondecreasing sublinear speedup
  - \text{prec} - precedence constraints
PROC-TIME-NON-INCREASING
The processing time $p(l)$ of a moldable task $J$ is non-increasing in the number $l$ of the processors allotted to it, that is,
$p(l) \leq p(l')$, for $l \geq l'$;
Common Assumptions - NdSub

- **PROC-TIME-NON-INCREASING**
  The processing time $p(l)$ of a moldable task $J$ is non-increasing in the number $l$ of the processors allotted to it, that is, $p(l) \leq p(l')$, for $l \geq l'$.

- **WORK-NON-DECREASING**
  The work $W(l) = w(p(l)) = lp(l)$ of a moldable task $J$ is non-decreasing in the number $l$ of the processors allotted to it, that is, $W(l) \leq W(l')$ for $l \leq l'$. 
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**PROC-TIME-STRICLTY-DECREASING**
The processing time $p(l)$ of a moldable task is strictly decreasing in the number $l$ of the allocated processors: $p(l) < p(l')$, for $l > l'$. 
SPEEDUP-CONCAVE "The first restriction is that all speedup functions are concave at least between 0 and the processor number $\hat{p}$ where the maximal speedup is reached." [SS12]
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WORK-CONVEX-PROC-TIME

The work function $w(p(l))$ is convex in the processing time $p(l)$. 
assumptions
1. PROC-TIME-NON-INCREASING
2. WORK-NON-DECREASING

decouple scheduling problem

allotment problem (MT-ALLOTMENT)
- Skutella’s linear relaxation of discrete time-cost trade-off problem (DTCT)
- 2 approximation

mapping problem (MT-MAKESPAN)
- Graham’s list scheduling for parallel tasks
  - earliest possible task first
- the proof is the trick here
- construct a heavy path in the transitive closure of the DAG

approximation ratio: $3 + \sqrt{5} \approx 5.23606$
same assumptions as [LTW02]

1. PROC-TIME-NON-INCREASING
2. WORK-NON-DECREASING

- provide linear program formulation
- kept mapping function
- the trick here is to find the right rounding parameter $\rho$
  - round fractional solution to feasible allotment
- approximation ratio: $\approx 4.730598$
assumptions:

1. PROC-TIME-NON-INCREASING
2. WORK-NON-DECREASING
3. SPEEDUP-CONCAVE

allotment: reformulate the LP

- essence: use a variable indicating that a job is (fractionally) allocated to \( l \) processors
- constraint: \( \sum_{l=1}^{m} x_{j,l} = 1 \)
- interesting here: at most two \( x_{j,l} \) are non-zero and adjacent

mapping step unchanged

approximation ratio: \( \approx 3.291919 \)
Related Work - Chen and Chu [CC13]

- an algorithm (heavily) based on JZ06
- assumptions
  1. PROC-TIME-NON-INCREASING
  2. WORK-NON-DECREASING
  3. WORK-CONVEX-PROC-TIME
- allotment:
  - precompute the work of each possible allocation for all tasks
  - use this information to add constraints to LP (which later help in the rounding step)
- mapping unchanged
- approximation ratio: $\approx 3.4142$
- then, adding
  1. PROC-TIME-STRictly-decreasing
  2. leads to approximation ratio: 2.9549
Reality 1
NAS PB - LU benchmark (4 sockets, 48 cores, AMD Opteron 6168)

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MPICH 3.0.4, 32x1 (process per node)
based on CPA by Radulescu and van Gemund [RvG01]

main purpose: schedule tasks with arbitrary speed-up functions

allotment solution, ingredients:
- consider only allocations which provide a relative runtime gain of $x\%$
- force task parallel execution of tasks
- iteratively add processors to tasks on critical path
- until $L_{CP} < W/m$

mapping solution
- similar to list scheduling approach of Lepère, Trystram, Woeginger
- but prioritize tasks by bottom level (length of path to sink)
- but consider packing of tasks if estimated completion time is not increased (binary search to find possibly smaller allotment)
General Questions

- How good are "frequently cited" heuristics (e.g., CPA) compared to approximation algorithms?
- How fast are current LP solvers (e.g., CPLEX) for solving "practically relevant" problems?

<table>
<thead>
<tr>
<th>algorithm</th>
<th>allocation</th>
<th>mapping</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPA13</td>
<td>$O(nm(n + e))$</td>
<td>$O(n(\log n + m \log m) + e)$</td>
</tr>
<tr>
<td>JZ06</td>
<td>$O(LP(mn, n^2 + mn))$</td>
<td>$O(mn)$</td>
</tr>
<tr>
<td>JZ12</td>
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<td>$O(mn)$</td>
</tr>
<tr>
<td>Chen13</td>
<td>$O(LP(mn, n^2 + mn) + mn)$</td>
<td>$O(mn)$</td>
</tr>
</tbody>
</table>

- let us fulfill all assumptions
  - PROC-TIME-NON-INCREASING, WORK-NON-DECREASING,
    PROC-TIME-STRICLTY-DECREASING, SPEEDUP-CONCAVE,
    WORK-CONVEX-PROC-TIME
Experimental Setup

Platform → DAG → Scheduler Simulation (Scala) → LP Instance Creator (Python) → Linear Program (GNU MathProg) → GLPK/GLPSOL

Solution to allocation problem → CPLEX → lp_solve → GLPK/GLPSOL → rounded solution
Simulation Results - Distribution of Makespans

Performance ratio \( \hat{\mu} = \frac{C_{\text{max}}}{LB} \)
Scalability

- Intel i7-3770 @ 3.40 GHz, 4 cores / 8 hardware threads
- CPLEX Studio 12.5.1 Linux x86-64
  - LP programs use all 4 cores
But can I be sure that the results are correct?
\[ \begin{align*}
\text{min} \quad & C \\
\text{such that} \quad & 0 \leq C_j \leq L \\
& C_j + x_k \leq C_k \\
& x_j \leq C_j \\
& x_j \leq p_j(1) \\
& x_{ji} \leq x_j \\
& 0 \leq x_{ji} \leq p_j(i) \\
& x_{jm} = p_j(m) \\
& \hat{w}_j(x_j) = \sum_{i=1}^{m} \bar{w}_{ji}(x_{ji}) \\
& P = \sum_{j=1}^{n} p_j(1) \\
& \sum_{j=1}^{n} \hat{w}_j(x_j) + P \leq W \\
& L \leq C \\
& W/m \leq C \\
& \bar{w}_j(x_{jm}) = 0 \\
& \hat{w}_{ji}(x_{ji}) = [W_j(i + 1) - W_j(i)] \frac{p_j(i) - x_{ji}}{p_j(i)} \\
\end{align*} \]
• LP of Chen and Chu also misses same constraint
• for assumption PROC-TIME-STRICTLY-DECREASING paper provides another rounding procedure leading to smaller bound
  • problem: rounding procedure described in paper for strictly decreasing function produced very large $C_{\text{max}}$'es (why?)
• run-time of this LP was huge
  • reason: LP uses a possibly different set of processor allocations for each moldable task (many more constraints)
Summary of Problems Occurred

- missing constraints in linear programs
  - very time-consuming to detect (at least for me)
- the problem of precision: floats/double
  - "64 Bit is finite"
  - problem generator
    - choose the execution time of tasks (for 1 proc) randomly and apply some strong strong scaling function
- 0.00000000000001345 \nless\n 0.00000000000001345

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- many sources of errors
  - me
  - DAG generator
  - platform generator
  - predictor for execution time of moldable tasks
  - translation of linear program in mathematical notation to AMPL/MathProg
  - parsing results from LP solver
  - implementation of mapping algorithm / simulator
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  - translation of linear program in mathematical notation to AMPL/MathProg
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  - implementation of mapping algorithm / simulator
- consequence: debugging is a nightmare
Summary - What Did I Miss in this Study?

- help from theoreticians
- public database with codes (e.g., LPs)
- set of benchmarks
  - relevant problems (for DAGs of moldable tasks)
  - possibly optimal solutions for small instances
  - implementations of algorithms (give me the sources)
  - some example schedules produced by algorithms
- wish list (from the view of a practitioner)
  - Complexity results for scheduling problems
    - [http://www.informatik.uni-osnabrueck.de/knust/class/](http://www.informatik.uni-osnabrueck.de/knust/class/)
    - add implementation and test cases
- my source code is available upon request
[CC13] Chi-Yeh Chen and Chih-Ping Chu.  

[Dro09] Maciej Drozdowski.  
*Scheduling for Parallel Processing*.  

An Approximation Algorithm for Scheduling Malleable Tasks under General Precedence Constraints.  

Scheduling malleable tasks with precedence constraints.  

Approximation Algorithms For Scheduling Malleable Tasks Under Precedence Constraints.  