AREA-Oriented DAG-Scheduling: A Preliminary Assessment

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An Emerging Challenge for HPC: ***Dynamically Heterogeneous Computing Platforms***

The constituent *"workers"* in many modern computing platforms

— e.g., *clouds*, or *grids*, *desktop grids*, *volunteer computing projects*

provide computing power at rates that cannot be known reliably $a\ priori$.

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The platforms exhibit *DYNAMIC HETEROGENEITY*

Is HPC achievable for Dynamically Heterogeneous platforms?

When jobs have *interchore dependencies* (modeled as DAGs)—

how can we cope with the *temporal unpredictability* of workers?

Is HPC achievable for Dynamically Heterogeneous platforms?

"When jobs have *interCHORE dependencies*"

We use the *granularity-neutral* term "chore" to capture "jobs," "tasks," etc., of arbitrary granularities.

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This is not always achievable!

Many DAGs do not admit schedules that *always* maximize the number of eligible chores.

BUT WE CAN ALWAYS

MAXIMIZE THE AVERAGE NUMBER OF ELIGIBLE CHORES.

DAG-Schedules and Their AREAs

A (computation-) DAG $\mathcal{G}$ represents a computational job.

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A (computation-) DAG $\mathcal{G}$ represents a computational job.

- Each $node$ of $\mathcal{G}$ is a $chore$
- Each arc of $\mathcal{G}$ is an $interchore\ dependency$

  Arc $(u \rightarrow v)$ means that chore v cannot be executed before chore u

  Chore u is a ***parent*** of chore v in $\mathcal{G}$

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A *schedule* Σ for $\mathcal{G}$ is a rule for selecting the next *eligible* chore to execute

(We measure time in an event-driven manner)

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Thus, Σ is a *topological sort* of $\mathcal{G}$.

DAG-Schedules and Their AREAs

Chore v of DAG $\mathcal{G}$ is *eligible (for execution) at step* t

if all of v's parents have been executed by step t

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$$AREA(\Sigma) \stackrel{\text{def}}{=} E_\Sigma(0) + E_\Sigma(1) + \cdots + E_\Sigma(N_{\mathcal{G}})$$

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(“AREA” invokes the analogy with Riemann sums as approximations of integrals)

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$$\approx\approx\approx\approx\approx\approx\approx\approx\approx$$

The (general version of the) AREA-MAX problem

GIVEN DAG $\mathcal{G}$, FIND A SCHEDULE Σ WITH MAXIMAL $AREA(\Sigma)$

is $\mathbf{NP}$-complete

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≈≈≈≈≈≈≈≈≈

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GIVEN DAG $\mathcal{G}$, FIND A SCHEDULE Σ WITH MAXIMAL $AREA(\Sigma)$

is $\mathbf{NP}$*-complete*

—via reduction from Minimum Weighted Completion Time

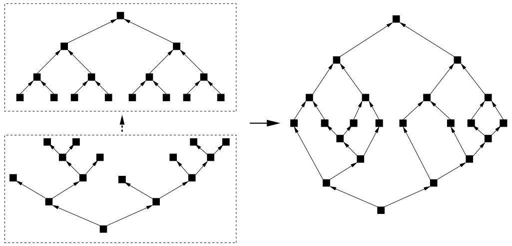
$\boxed{\textbf{Responding to the NP-completeness of AREA-MAX}}$

1. $\boxed{\textit{Efficient AREA-maximizing schedules for many DAG-families}}$

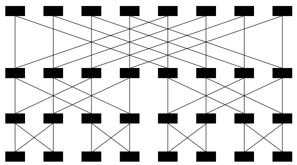
2. Efficient heuristics that “seem” to work well

Efficient AREA-maximizing schedules for many DAG-families

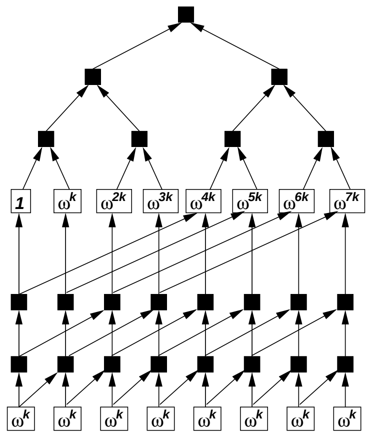
Expansive-Reductive (ER) DAGs



Convolutional DAGs



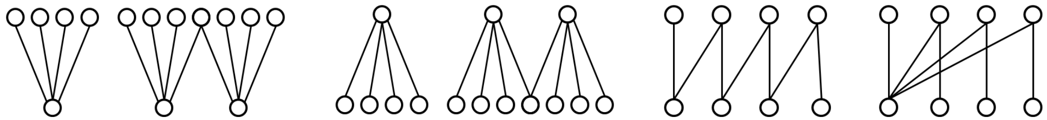
Compositions of ER, Convolutional DAGs



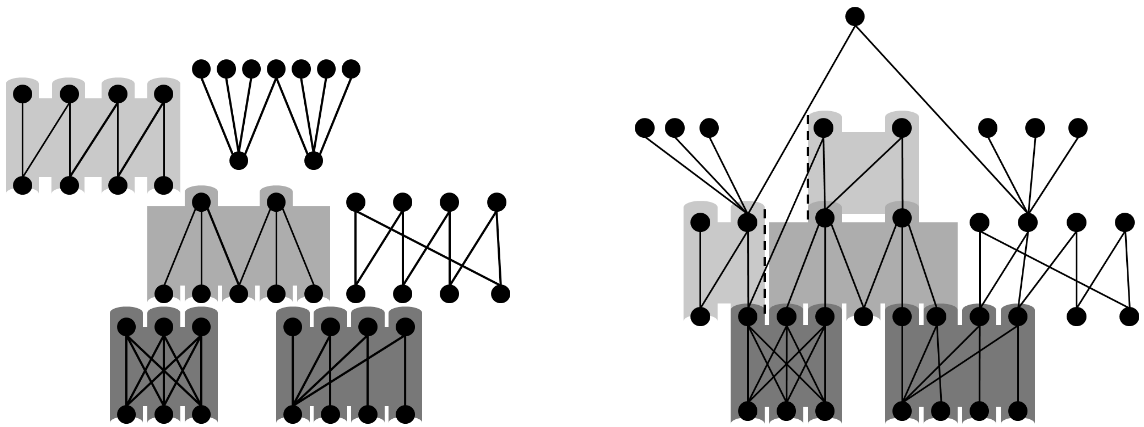
Efficient AREA-maximizing schedules for many DAG-families

LEGO-DAGs — a family of families of significant DAGs

Some bipartite building-block DAGs. (All arcs point upward.)



Composing building blocks (left) into a *LEGO*-DAG (right)

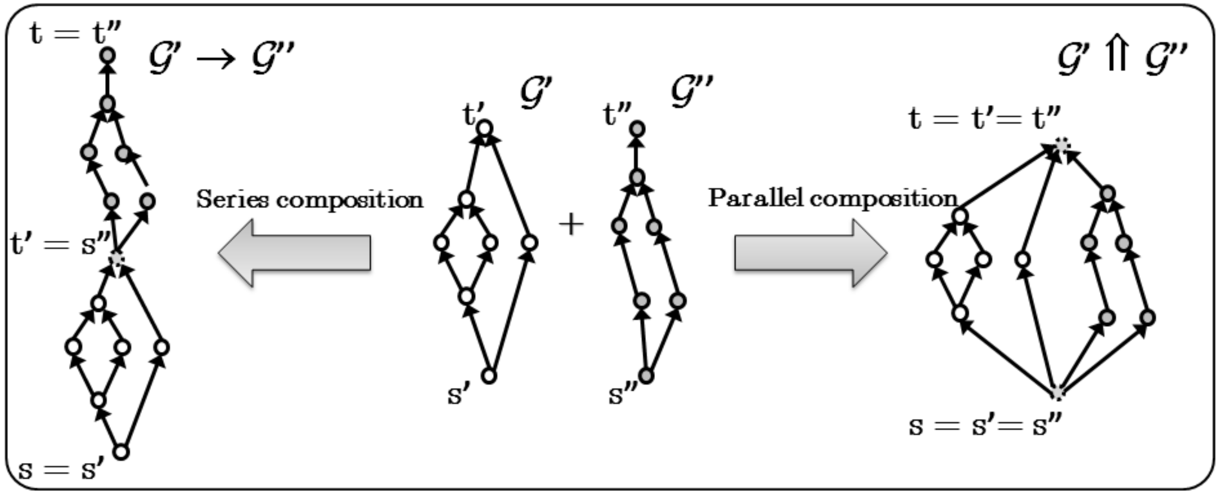


Efficient AREA-maximizing schedules for many DAG-families

A *REALLY* important special family: *Series-Parallel DAGs* (*SP-DAGs*)

- model multi-threaded computations
- lead to a good DAG-scheduling heuristic

The defining compositions that produce *SP-DAGs*



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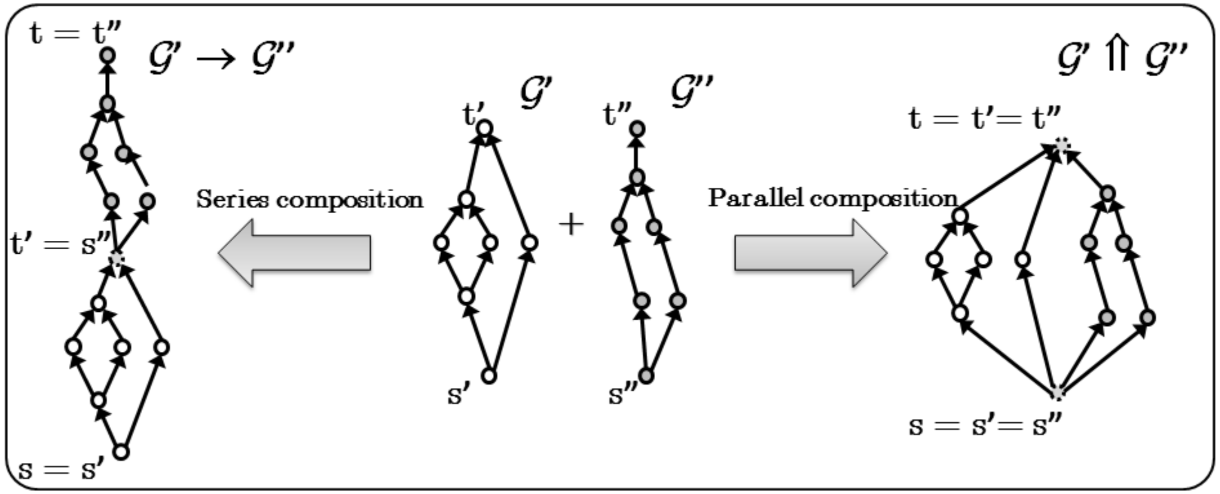


One can find an AREA-maximizing schedule for any SP-DAG in quadratic time

Responding to the NP-completeness of AREA-MAX

1. Efficient AREA-maximizing schedules for many DAG-families

2. *Efficient heuristics that "seem" to work well*

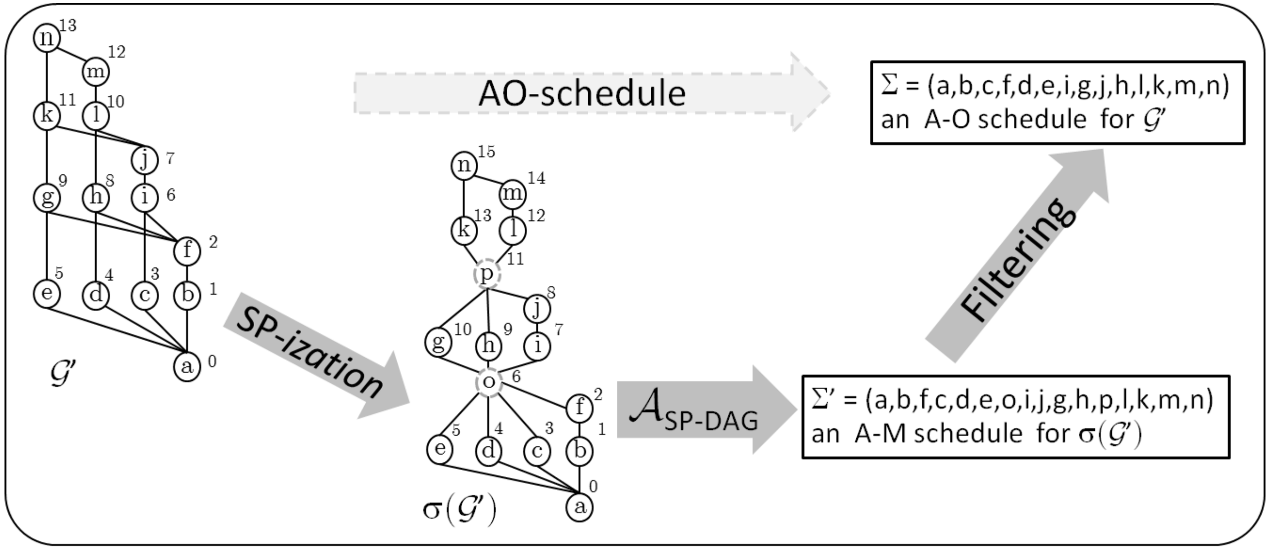
Converting an arbitrary DAG to a SP-DAG

In quadratic time, one can convert any DAG $\mathcal{G}$

to an SP-DAG $\sigma(\mathcal{G})$ (via an "SP-ization")

that has "roughly as much" parallelism as $\mathcal{G}$

A sample SP-ization. (Note "additional" [synchronizing] chores)



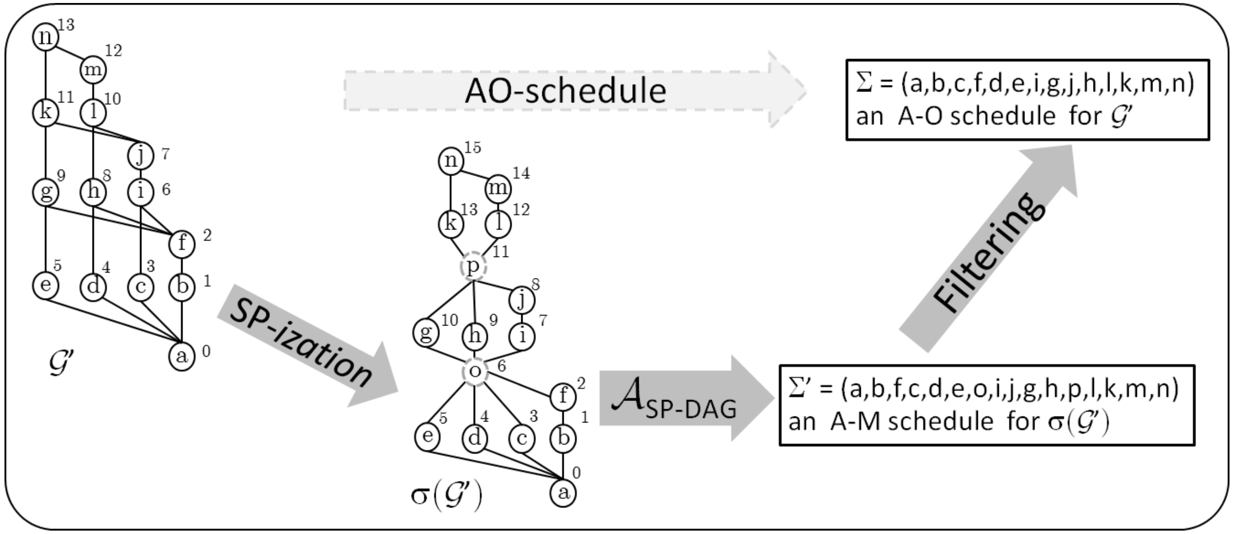
The efficient AOSPD heuristic —that "seems" to work well

The **AOSPD** DAG-scheduling heuristic

$\mathcal{G}$ is the DAG that you want to schedule with large AREA

If $\mathcal{G}$ is an SP-DAG
then use our AREA-maximizing SP-DAG scheduler
else

1. Transform $\mathcal{G}$ to the SP-DAG $\sigma(\mathcal{G})$ using a prescribed SP-izer
2. Use our AREA-maximizing SP-DAG scheduler to schedule $\sigma(\mathcal{G})$
3. "Filter" the resulting schedule to eliminate the "additional" chores



$\boxed{\textbf{EXPERIMENTAL SECTION}}$

Assessing the A-O Paradigm's Impact — via the AOSPD heuristic

Our assessment performs the following tests:

- AOSPD's schedules' MAKESPANs vs. other "oblivous" schedulers'
- AOSPD's schedules' AREAs vs. other "oblivous" schedulers'
- AOSPD's schedules' AREAs vs. true AREA-Maximizing schedules
  —for DAGs whose A-M schedules we know how to generate efficiently
- Test the hypothesis that larger AREA means smaller MAKESPAN
  —Is there a positive correlation?

The Competing “Oblivious” Schedulers

- The **FIFO** scheduler:
  Stores newly eligible chores in a fifo queue
  —very lightweight, not very effective

- The **Static-Greedy** scheduler:
  Stores newly eligible chores in a MAX-priority queue
  ordered by *outdegree*
  —much less lightweight, more effective

- The **Dynamic-Greedy** scheduler:
  Stores newly eligible chores in a MAX-priority queue
  ordered by *yield* (number of chores they would render eligible)
  —rather heavyweight, quite effective (*one-step optimal*)

The Experimental Protocol

- **The MAKESPAN experiment**

  Compare AOSPD's **performance to** heuristic H's **via the ratio** $T(H) \div T(AO)$

  – Chore execution-times are distributed normally (positive half):
  mean $= 1$;   std-dev $\in \{0.1, 0.5\}$

  – The number of available workers at step (c_t) is distributed exponentially with rate parameter λ:
  $P[c_t = x] = \lambda e^{-\lambda x}$; $\lambda \in \{2^{-k} | k \in [0..7]\}$

- **The AREA experiment**

  Compare AOSPD's **performance to** heuristic H's **via the ratio** $AREA(AO) \div AREA(H)$

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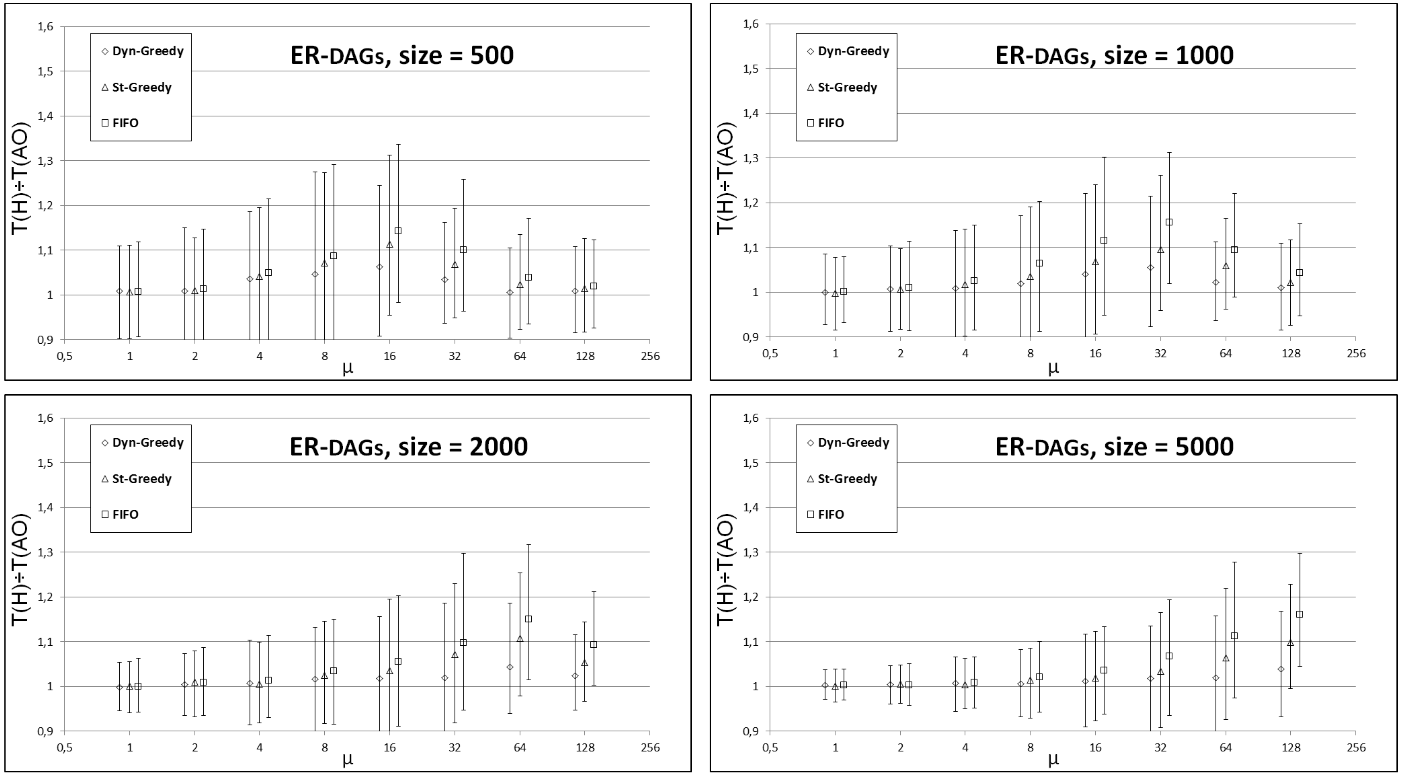
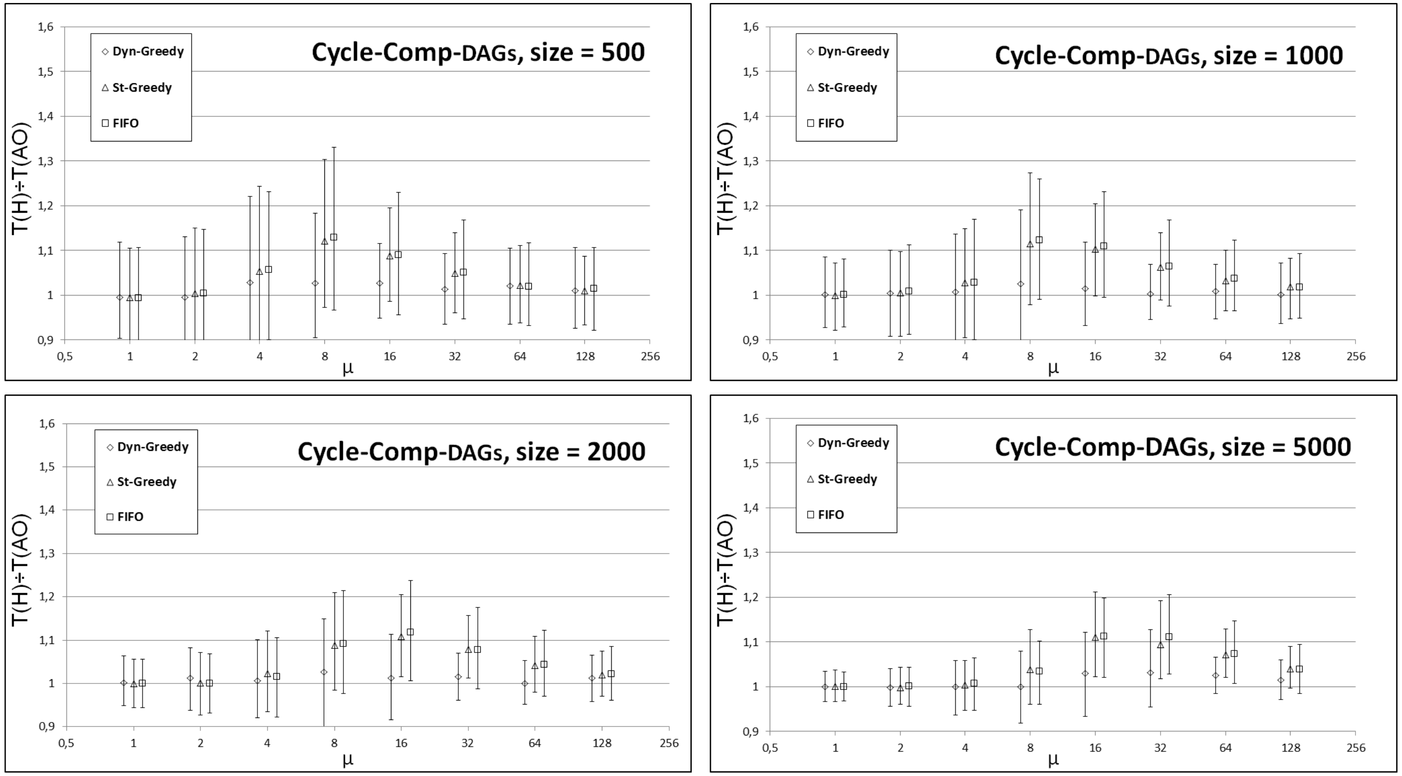
RECALL THAT $\begin{cases} \text{SMALLER MAKESPAN IS BETTER} \\ \text{BIGGER AREA IS BETTER} \end{cases}$

\# EXPERIMENTAL RESULTS

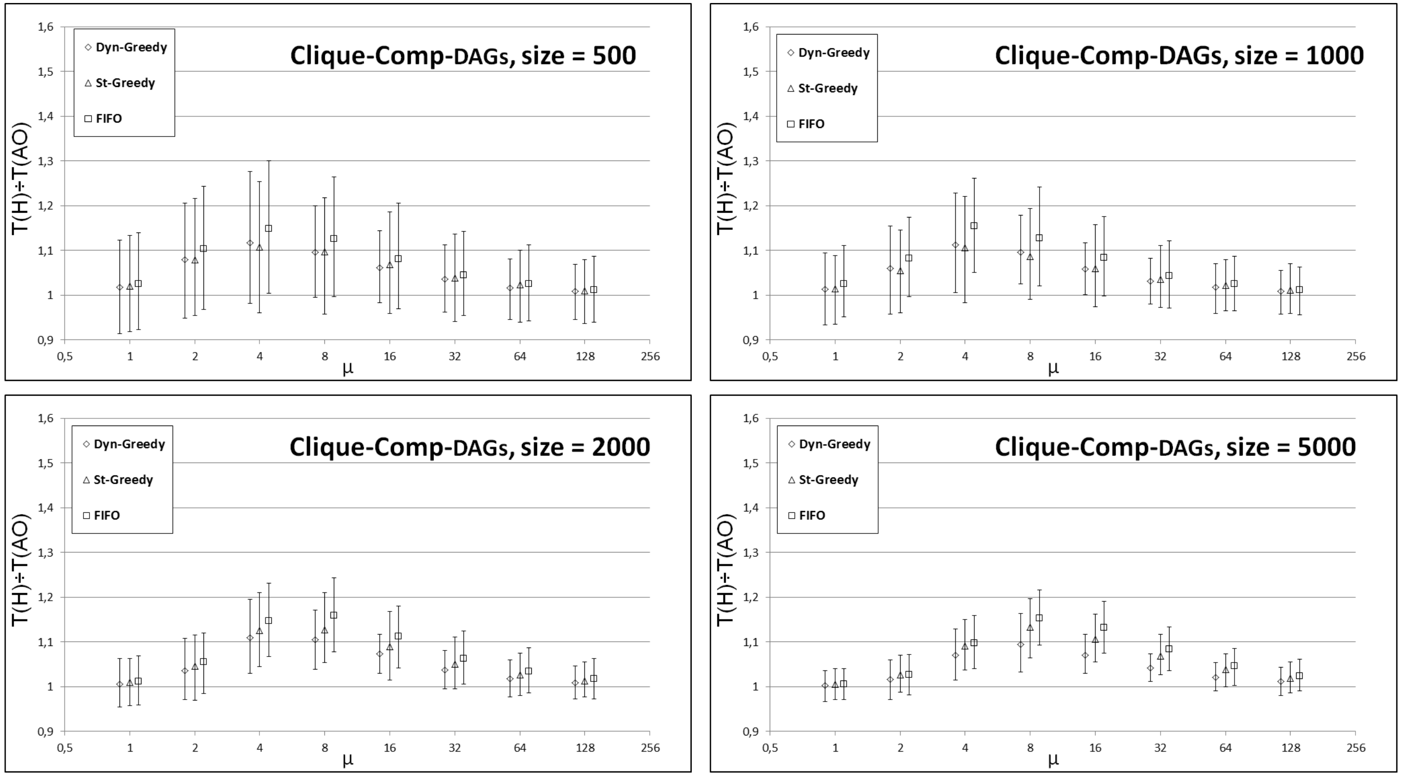
AO's schedules' MAKESPANs vs. Competitors': Expansive-Reductive (ER) DAGs



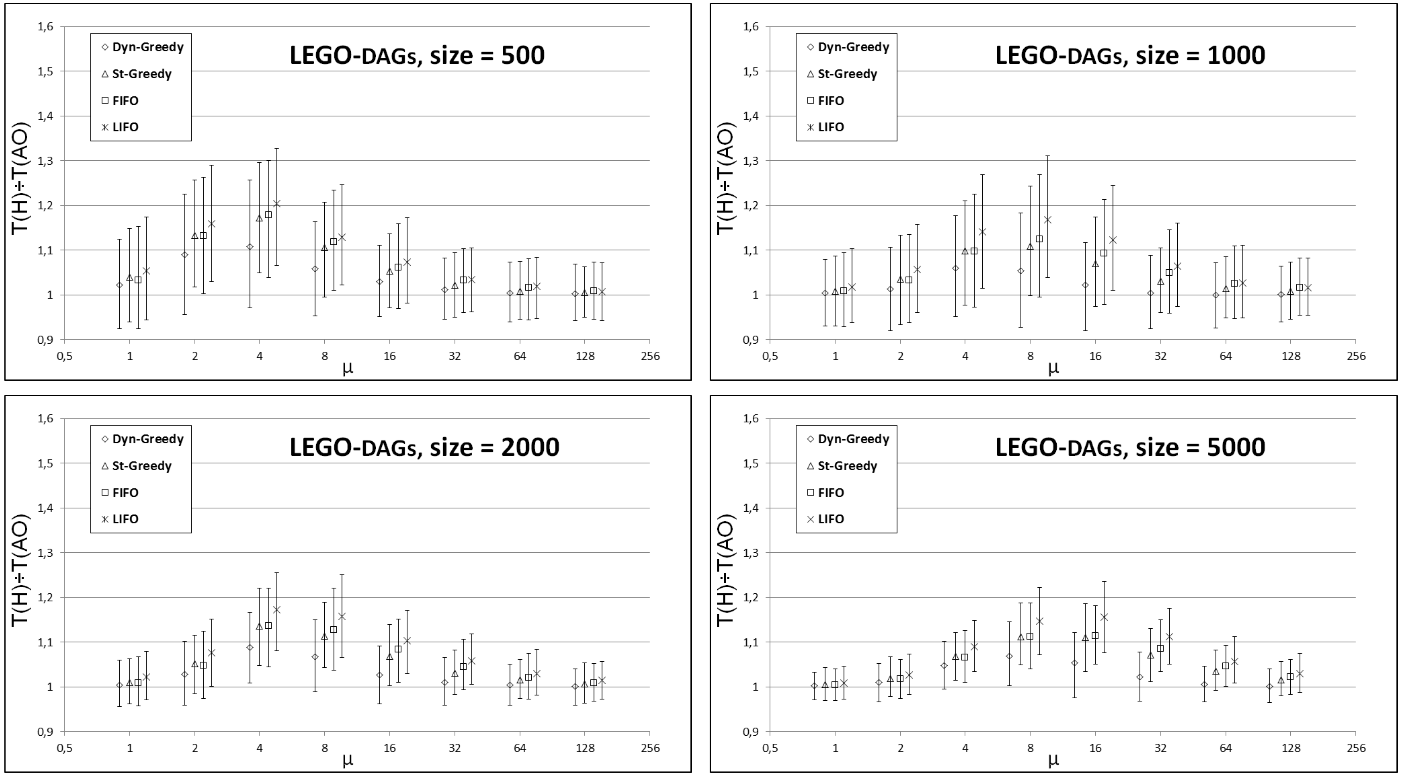
AO's schedules' MAKESPANs vs. Competitors': Cycle-composition DAGs



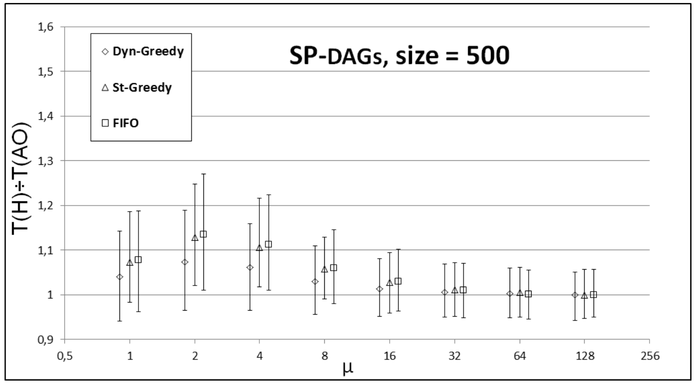
AO's schedules' MAKESPANs vs. Competitors': Clique-composition DAGs

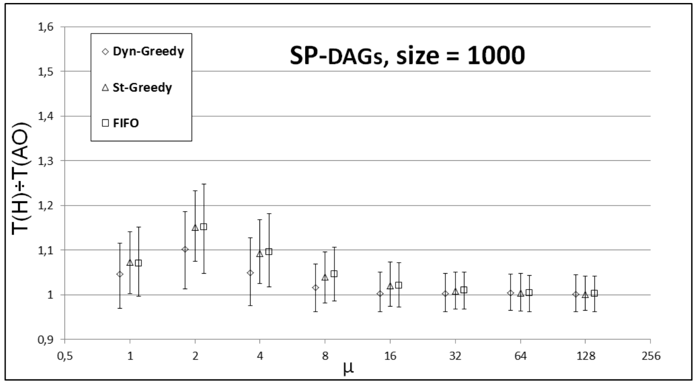


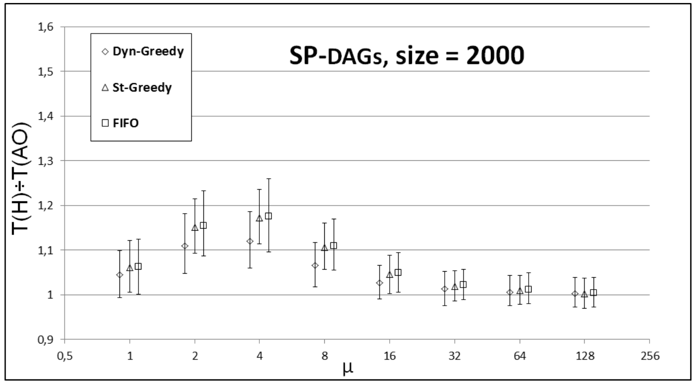
AO's schedules' MAKESPANs vs. Competitors': <u>LEGO-DAGs</u>

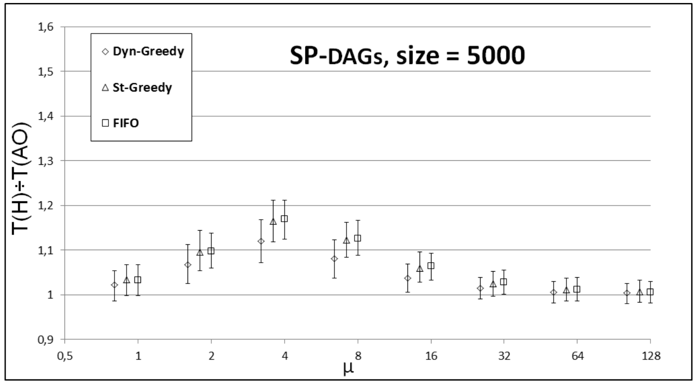


AO's schedules' MAKESPANs vs. Competitors': <u>Series-Parallel DAGs</u>



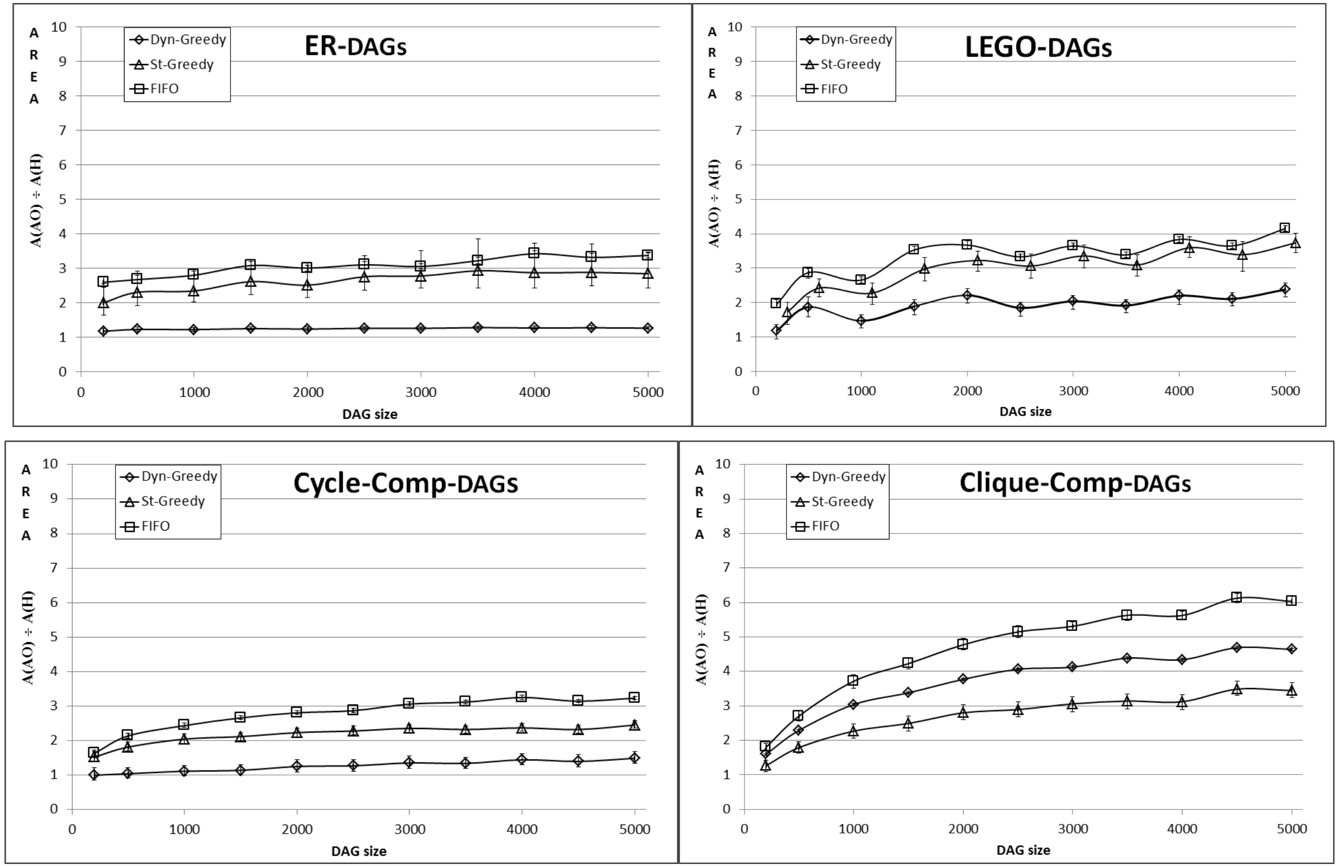






AO's schedules' AREAs vs. Competitors': ER-DAGs, Cycle-composition DAGs, Clique-composition DAGs, LEGO-DAGs

Mean ratios and ranges: $AREA(AO) \div AREA(H)$



AO's schedules' AREAs vs. Competitors': Series-Parallel DAGs

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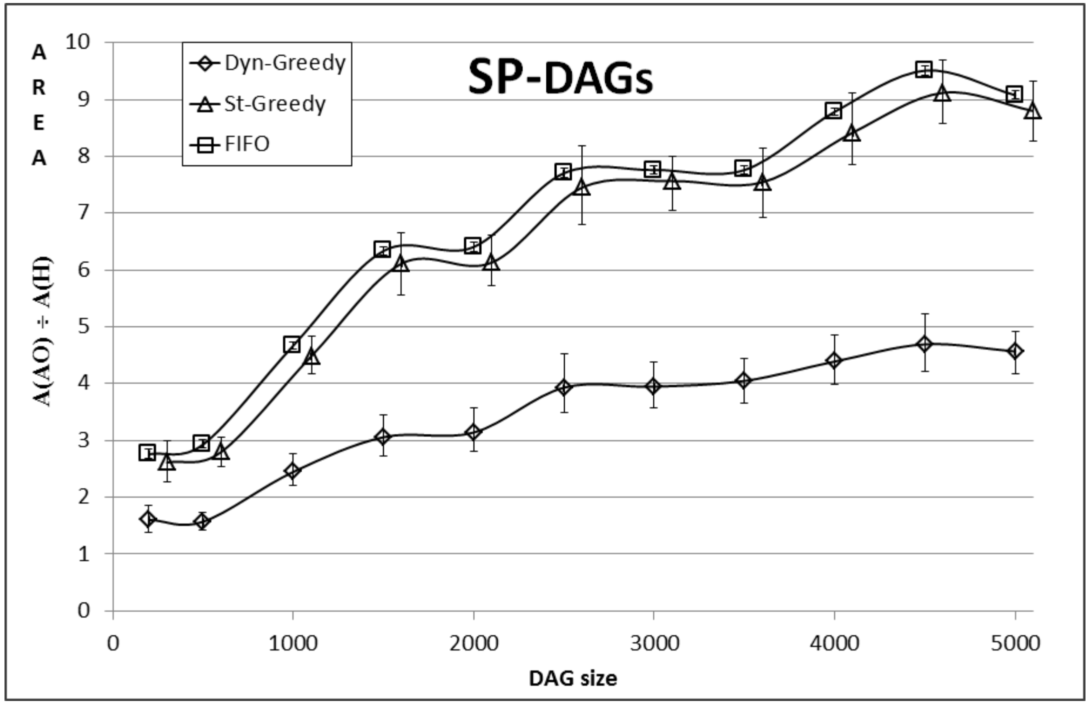
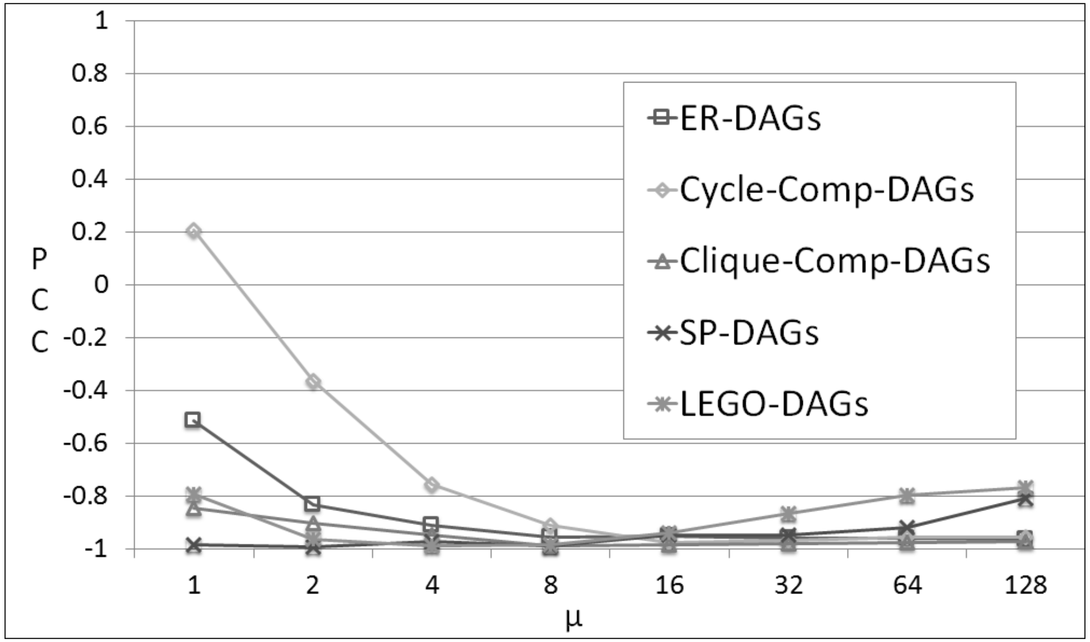


Does larger AREA mean smaller MAKESPAN?

Correlating AREA and MAKESPAN for various DAG-familiess

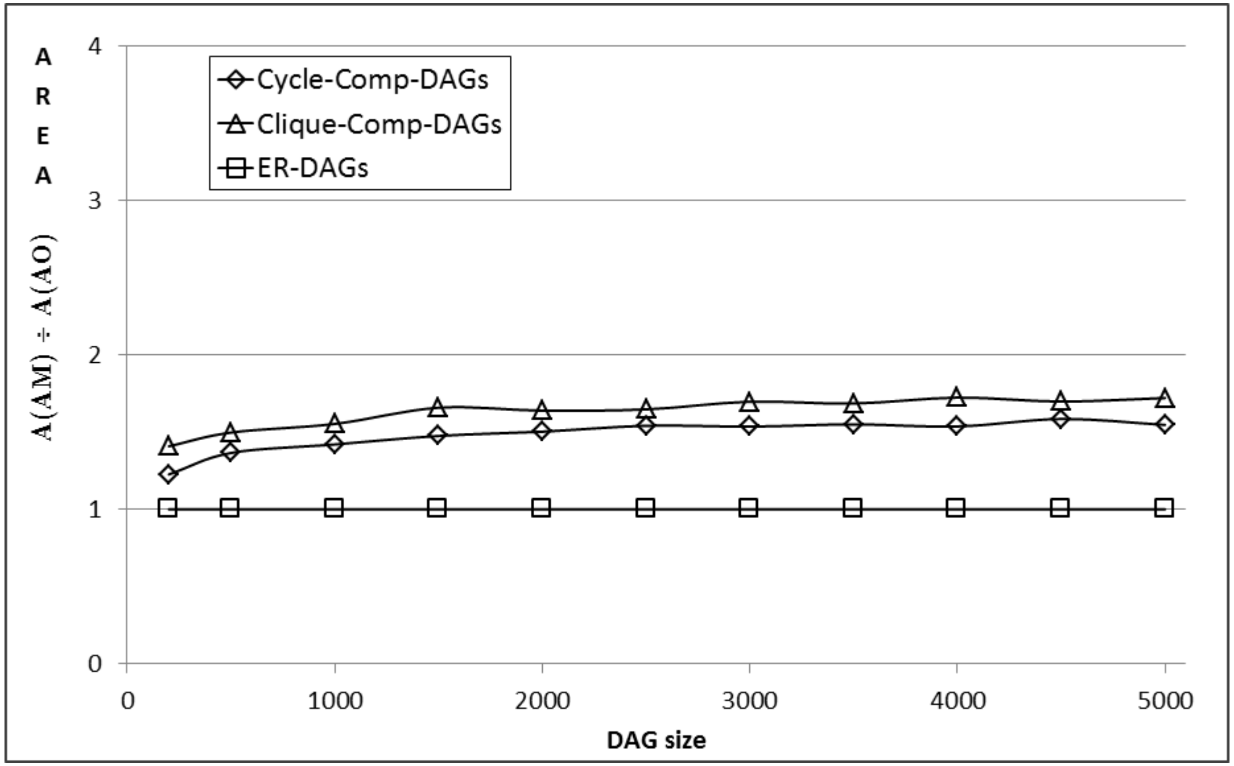


- μ is the "arrival rate" of available processors
- PCC is the *Pearson Product-Moment Correlation Coefficient* — measures correlation via both <u>strength</u> and <u>direction</u>

Comparing AREA-Maximization vs. AREA-Orientation via schedule AREA

Comparing the <u>AREAs</u> of A-M and A-O schedules

—via the ratio $AREA(AM) \div AREA(AO)$



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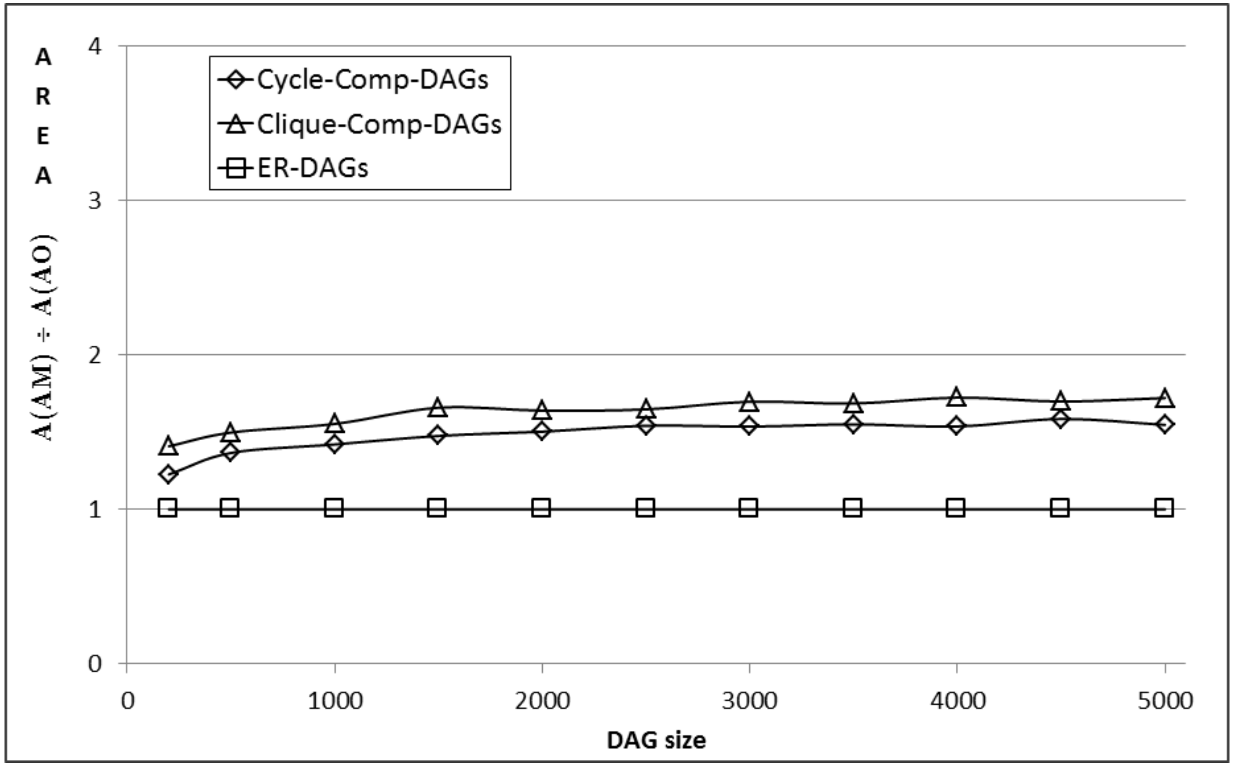


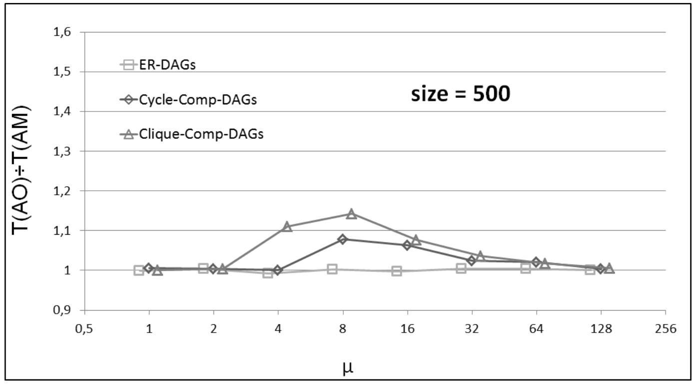
Observe that $AREA(AM) \ \leq \ 2 \times AREA(AO)$.

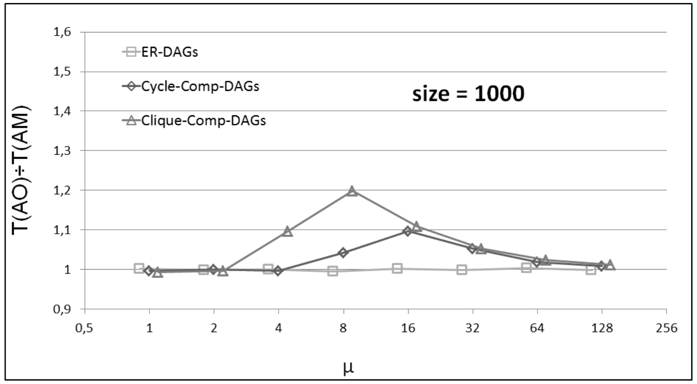
IS THIS ALWAYS TRUE?

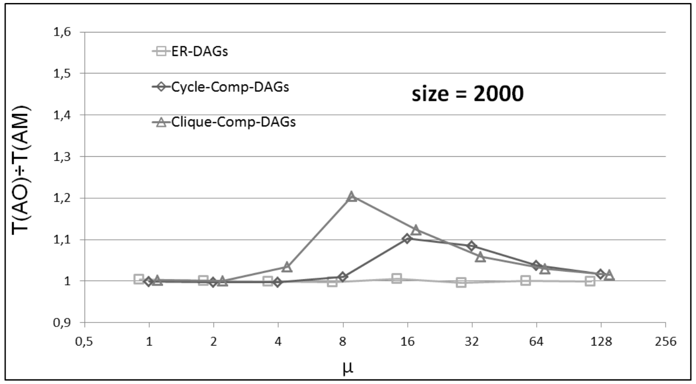
Comparing AREA-Maximization vs. AREA-Orientation via schedule MAKESPAN

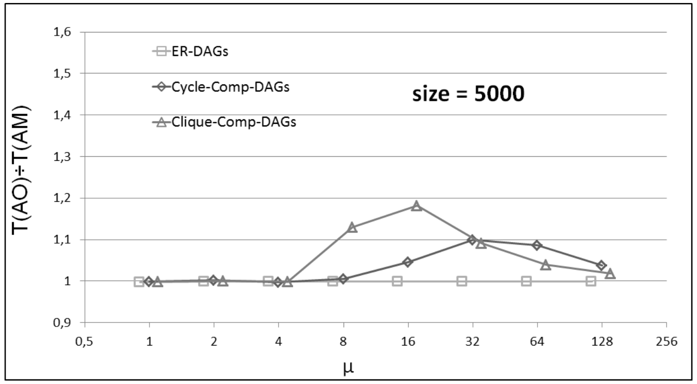
Comparing the MAKESPANs of A-M and A-O schedules

—via the ratio $T(AO) \div T(AM)$





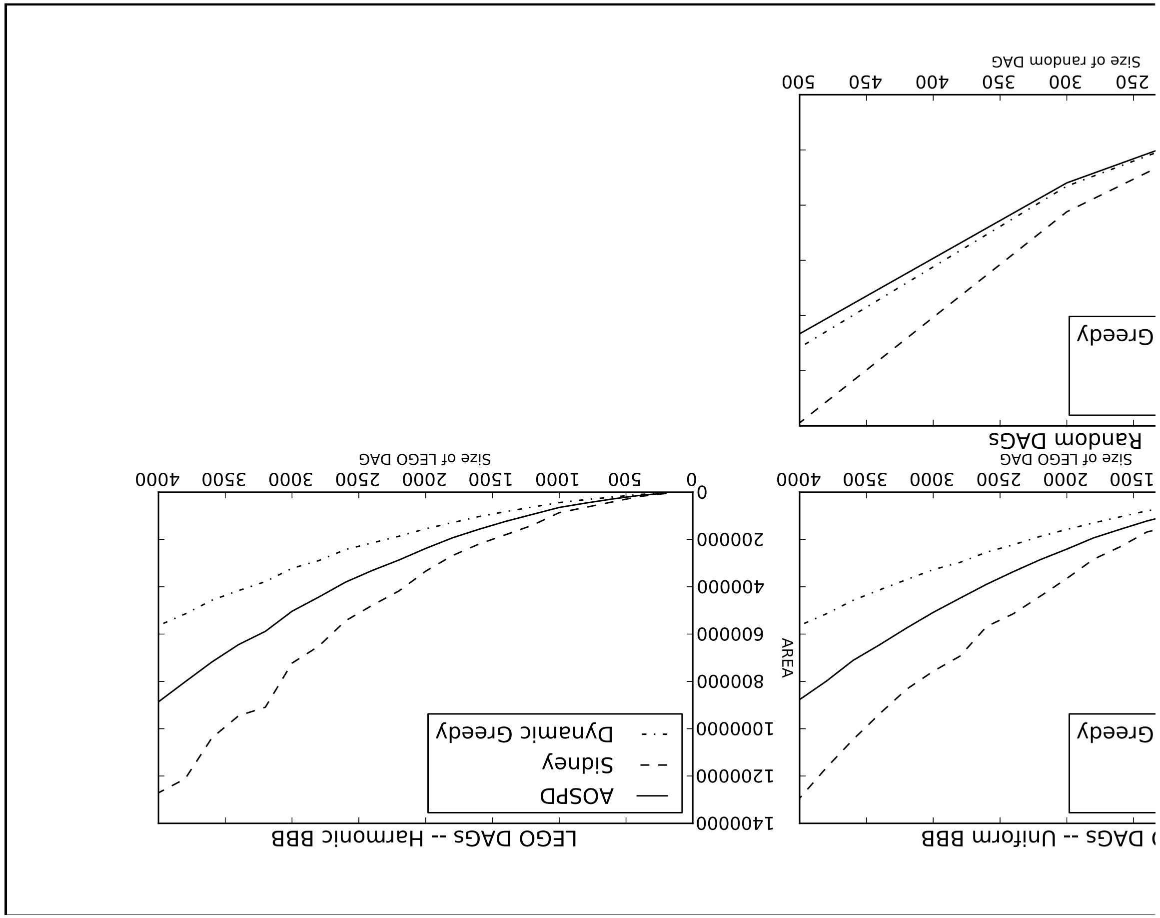


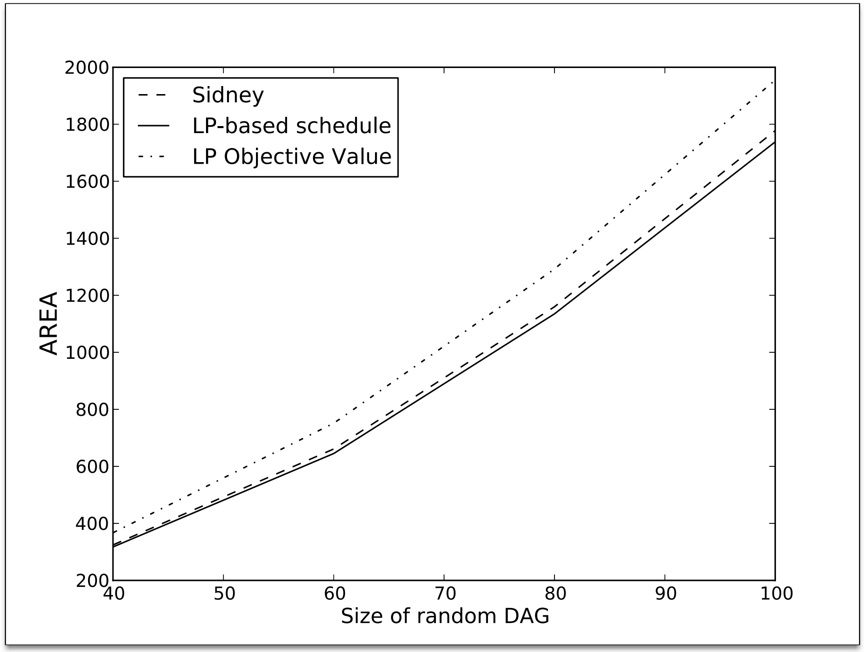


The SIDNEY Scheduling Heuristic

- Based on the Sidney decomposition of DAGs
- Experiments show that SIDNEY's schedules
  - have AREAs significantly larger than other heuristics'
  - have AREAs within 85% of optimal on randomly generated DAGs

The AREA-qualities of SIDNEY's schedules.




Sidney
LP-based schedule
LP Objective Value
AREA
Size of random DAG
2000
1800
1600
1400
1200
1000
800
600
400
200
40
50
60
70
80
90
100