Efficient and Robust Allocation Algorithms in Clouds under Memory Constraints

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Provider-side allocation

Typical Cloud Computing scenario

- A number of clients submit services to a provider

  Think of services as commercial websites
- Services have hardware requirements (CPU, memory, I/O, ...)
- Clients express demands: for each service, enough CPU must be available (serve enough requests per second)

Optimization

- Several services can be allocated to the same Physical Machine (PM)
- Optimize energy and resource usage: consolidation
- Allocation of services onto PMs:

  MultiDimensional Bin Packing Problem

Introducing machine failures

Data centers are large

- Failures will happen
- Our approach: over-provisioning – allocate additional capacity
- Clients express a reliability requirement

expressed as a probability, or as a cost penalty

Assumptions

- Static setting over a given time period (between two migration phases):
  - Compute an allocation at the start of the period
  - During the time period, some machines fail
  - Services should still be running (enough instances) at the end
- Machines fail independently with probability f

Problem formulation

Notations

- Identical machines, with capacity C and memory M
- ns services: demands d_i, reliability requirement r_i, memory m_i
- **Variables** $A_{i,j}$: CPU capacity for service i on machine j
- $Alive\_cpu_i = \sum_{j=1}^{m} is\_alive_j \times A_{i,j}$, where $is\_alive_j = 0$ with prob. f

Formulation

$$\text{minimize } m \text{ s.t. } \begin{cases} \forall i, \mathbb{P}\left(Alive\_cpu_i < d_i\right) < r_i \\ \forall j, \sum_{A_{i,j}>0} m_i \leq M \\ \forall j, \sum_{i=1}^{ns} A_{i,j} \leq C \end{cases}$$

Two-step algorithm

1. Load Balancing
   Relaxed formulation to compute resource usage (CPU, memory) for each service so as to satisfy reliability constraints
2. Packing
   Use column generation techniques to obtain a feasible packing

First step: focus on reliability

Approximation by normal distribution

Given an allocation $A_{i,j}$, computing $\mathbb{P}(Alive\_cpu_i < d_i)$ is #P-complete.
(as hard as counting the # of solutions to a knapsack problem)

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$\mathbb{P}\left(Alive\_cpu_i < d_i\right) < r_i$ is approx. by $\sum_{j=1}^{m} A_{i,j} - B_i\sqrt{\sum_{j=1}^{m} A_{i,j}^2} \geq K_i$

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Relaxed formulation (lower bound)

minimize m s.t.

$$\begin{cases} \forall i, \sum_{j=1}^{m} A_{i,j} - B_i \sqrt{\sum_{j=1}^{m} A_{i,j}^2} \geq K_i \\ \sum_j \sum_{A_{i,j}>0} m_i \leq mM \\ \sum_j \sum_{i=1}^{ns} A_{i,j} \leq mC \end{cases}$$

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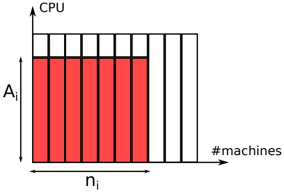
Homogeneous allocations are dominant ($A_{i,j} = A_i$ or 0)



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Relaxed formulation (lower bound)

$$\begin{cases} \forall i, \sqrt{n_i} > B_i \\ \sum_i n_i m_i \leq mM \\ \sum_i \frac{K_i}{1-\frac{B_i}{\sqrt{n_i}}} \leq mC \end{cases}$$

Can be solved for fractional n_i.

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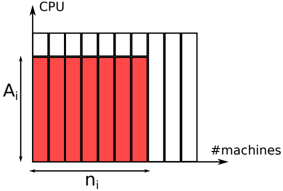


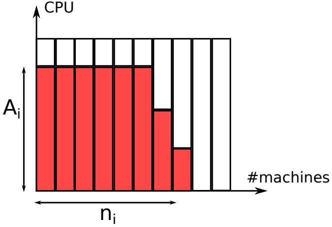
Second step: packing

- Solve the homogeneous pb to get "ideal" allocations n_i, A_i
- "Pack" them onto individual machines

Remember the approximate formulation

$$\text{minimize } m \text{ s.t. } \begin{cases} \forall i, \sum_{j=1}^{m} A_{i,j} - B_i \sqrt{\sum_{j=1}^{m} A_{i,j}^2} \geq K_i \\ \forall j, \sum_{A_{i,j}>0} m_i \leq M \\ \forall j, \sum_{i=1}^{ns} A_{i,j} \leq C \end{cases}$$

"Splitting" a service always improves the first inequality



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Splittable bin packing with memory constraints

- Allocate $n_i A_i$ total capacity to service i
- Chunk of at most A_i on each machine
- Each chunk uses m_i memory
- At most M memory used and C capacity per machine

LP Formulation with configurations

Valid configuration $\mathcal{C}_c$

- $\forall i$, fraction $x_{i,c}$ of the maximum capacity A_i devoted to service $\mathcal{S}_i$
- c is valid iff $\sum_i x_{i,c} \leq C$ and $\sum_{x_{i,c}>0} m_i \leq M$
- **Variable** λ_c: number of machines with this configuration
- Almost full configurations are enough:
  $x_{i,c} = 1$ or 0 except for one service

Configuration formulation

$$\text{minimize} \sum_{c \in \mathcal{F}} \lambda_c \text{ st } \quad \forall i, \sum_{c \in \mathcal{F}} \lambda_c x_{i,c} \geq n_i$$

Column generation

$$\textbf{Primal:}\ \text{minimize} \sum_{c\in\mathcal{F}} \lambda_c \text{ st} \quad \forall i, \sum_{c\in\mathcal{F}} \lambda_c x_{i,c} \geq n_i$$

$$\textbf{Dual:}\ \text{maximize} \sum_{i} n_i p_i \text{ st} \quad \forall c \in \mathcal{F}, \sum_{i} x_{i,c} p_i \leq 1$$

Principle: iteratively augment the set of configurations $\mathcal{F}$

- Start with a small set of configurations
- Solve the primal, get a sub-optimal solution
- Find a violated constraint in the dual (splittable knapsack)
- Add this configuration to the set and loop

Splittable knapsack

Splittable knapsack problem

- Given A_i, m_i, p_i, M and C,
- Find J and x_i such that $\sum_{i \in J} m_i \leq M$, and $\sum_{i \in J} x_i A_i \leq C$
- So as to maximize $\sum_{i \in J} x_i p_i$

Properties

- Almost full configurations are dominant.
- NP-hard (from 2-Part)
- $O(MC)$ Dynamic Programming algorithm to solve optimally.

Summary

Two-step heuristic

- Compute a homogeneous lower bound n_i, A_i (binary search)
- Solve the packing problem
  - Start with a small set of configurations
  - Solve the primal, get a sub-optimal solution
  - Find a violated constraint in the dual (splittable knapsack)
  - Add this configuration to the set and loop

Experimental evaluation

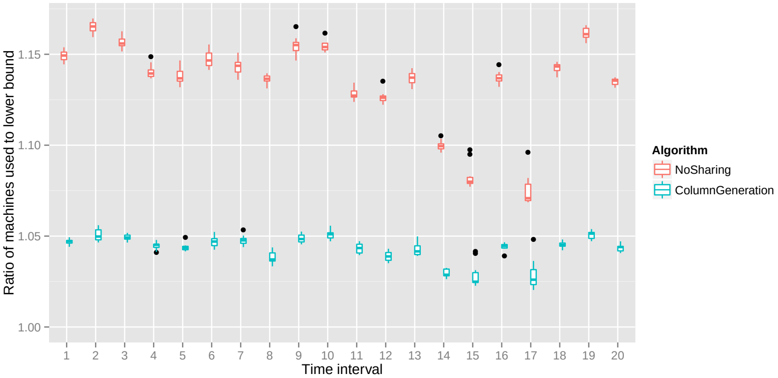
Setting

- Memory and CPU usage from public Google trace 150 jobs account for 90% of resource usage
- $r_i = 10^{-X}$, with $X \simeq \mathrm{U}(2, 8)$
- $f = 0.01$

NoSharing heuristic

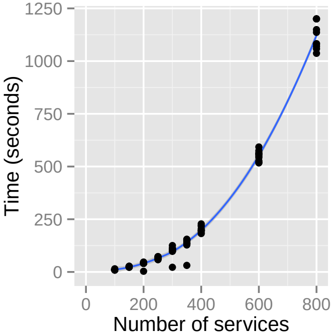
- Computes homogeneous allocations with $\forall i, A_i = C$
- Exactly one service per machine

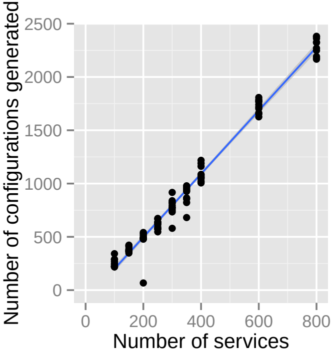
Experimental results: number of machines



Running time of ColumnGeneration: mean 14.6s, max 22.2s

Experimental results: execution time





Conclusions

Explored allocation problems with reliability constraints

- Binomial approximation allows a good lower bound
- Reformulations & column generation give good heuristics

Further directions

- Extensions: multi-dimensional (CPU + IO), heterogeneous machines, ...
- Migrations for dynamic setting
- Recycle the techniques for other problems